REGIONAL GROWTH IN A MULTIREGIONAL I-O FRAMEWORK[1]

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Abstract

Since the pioneering work of Wassilly Leontief, a remarkable amount of theoretical and empirical work has continuously supported Input-Output modelling. In particular, the peculiar structure of dynamic Input-Output models have originated, in numerous fields ranging from Mathematical Economics to System Theory, an abundance of contributes. This paper deals with the computational problem of managing regional growth within a dynamic multiregional Input-Output model. Starting from the basic matrices of technological, capital and trade coefficients, the regional components associated to a given group of regions are appropriately recognised and separated. A numerical example based on the Italian case, is also discussed.

An earlier version of this paper was presented at the Thirty-fourth Annual Meeting of Western Regional Science Association in San Diego and has benefitted from comments during the conference. The authors are grateful for suggestions rendered by the anonymous referees. We also wish to thank Prof. G. Hewings and P. R. Israilevich for their useful comments and suggestions.

1. INTRODUCTION

Recent theoretical and empirical evidences (Sommer and Hicks, 1993; Sonis et al., 1995) have demonstrated that significant changes in the growth process of national economies have to be addressed to local agglomerate processes. To this aim a large amount of research effort have been addressed to multiregional dynamic Input-Output models (Guccione et al., 1988; Sonis et al., 1993). The above scheme had to overcome both theoretical and empirical difficulties. Theoretical difficulties due to the nature of interregional linkages (Campisi and Nastasi, 1993) and the possibility of projecting forward the Leontief dynamic system (Campisi et al., 1993). From the empirical point of view, once the conventional data availability is accepted, the major problem is to identify the best level of details to incorporate the spatial linkages and feedback effects (Sonis et al., 1995). The aim of this paper is therefore to provide a workable tool to overcome the above difficulties. The proposed approach is based on the balanced multiregional balanced growth solutions provided by Campisi et al. (1991) and the decomposition technique for linear systems developed by Meyer (1989). The first scheme allows to evidence the capital accumulation process and multisectoral multiregional interactions which evolve within a general equilibrium contest. Workable assumption that each sector in the economy requires, directly or indirectly, either some current flow or some capital input from all the other sectors operating in the same and in the other regions is assumed. The Meyer scheme allows, on the other side, to manipulate, through decomposition, large scale systems as the described one. The advantage of integrating such tools mainly resides in the possibility of considering all the available linkages deriving from modern data sources in an effective manner. The proposed methodology, then, allows to condense in very limited amount of equations all the complex relations arising from detailed multiregional models. The only requirements are due to the computation of the dominant eigenvalue of a generalised Leontief inverse matrix whose dimension is given by the product of the number of sector by the number of regions and the presence of conventional round-off. These items are not serious problems also for conventional Personal Computers. On the contrary the computation of large scale eigenvectors is overcame by means of the computation of disaggregate components. Moreover, our research results allow to identify in terms of casual linkages, all the components in disaggregate systems. By summarising, in presence of a large scale multiregional I-O scheme, the proposed approach allows to maintain the features of very disaggregated schemes with limited storage requirements, give a straightforward interpretation of the components contributing to the growth process and enable to choose the level of analysis with a large amount of flexibility.

The paper is organised as follows. In section 2 and 3 the multiregional Input-Output model is introduced and its structural properties are analysed in detail. In section 4 a technique allowing the decomposition at regional level of the growth factors in the multiregional I-O model, is described. Moreover, in section 5 a brief description of the related computer program structure, is presented. In section 6, finally the procedure is applied to analyse the growth of the Italian economy on the basis of the 1985 biregional matrices of capital and technological requirements.

2. THE DYNAMIC MULTIREGIONAL INPUT-OUTPUT MODEL

In this section we present the structure of the multiregional dynamic Leontief model which will be utilised in the sequel as the basis for our analysis (Campisi et al., 1993). Consider an economic system subdivided into m spatially defined economies (regional economies) where time elapses with a succession of discrete periods. There are n productive sectors, each one producing only one commodity by means of only one linear production process lasting only one period. Every sector requires several resources to carry out its activity. Let a_{js}^{ir} be the current input of commodity i produced in region r used to produce one unit of commodity j in region s; notice that the subscripts and the superscripts denote, respectively, place of origin and destination):

$$a_{js}^{ir} = x_{js}^{ir}(k) / x_{js}(k); \quad (i,j=1,2,...,n); \quad (r,s=1,2,...,m) \tag{1}$$

where $x_{js}^{ir}(k)$ is the total current input from sector i in region r required in period k by sector j in region s and $x_{js}(k)$ is the total output produced in the same period by sector j in region s. Each interregional current input coefficient a_{js}^{ir} can be split into a regional coefficient a_{js}^{i}, which represents the current input from sector i, wherever located, needed per unit of output of sector j in region s, and an interregional trade coefficient t_{js}^{ir}, which represents the proportion of the total current input of i required by sector j in region s supplied by sector i in region r :

$$a_{js}^{ir} = t_{js}^{ir}\, a_{js}^{i}; \quad (i,j=1,2,...,n); \quad (r,s=1,2,...,m) \tag{2}$$

$$\text{where} \quad \sum_{r=1}^{m} t_{js}^{ir} = 1 \qquad \forall\ i, j, s.$$

Let b_{js}^{ir} be the stock of commodity i produced in region r used as capital good in the production of one unit of commodity j in region s :

$$b_{js}^{ir} = k_{js}^{ir}(k) / x_{js}(k); \quad (i,j=1,2,...,n); \quad (r,s=1,2,...,m) \tag{3}$$

where k_{js}^{ir} (k) is the total stock of commodity i produced in region r required as capital good in period k by sector j in region s. Each interregional capital input coefficient b_{js}^{ir} can be split into a regional coefficient b_{js}^{i}, which represents the capital input from sector i, wherever located, needed per unit of output of sector j in region s, and an interregional trade coefficient t_{js}^{ir}, which represents the proportion of the total capital input of i required by sector j in region s supplied by sector i in region r :

$$b_{js}^{ir} = t_{js}^{ir}\, b_{js}^{i}; \quad (i,j=1,2,...,n); \quad (r,s=1,2,...,m) \tag{4}$$

where the t_{js}^{ir}'s for commodity i used as capital input are the same interregional trade coefficients defined for commodity i used as current input for all i, j, r, s.

In order to facilitate the analysis the following assumptions are also imposed:

- a_{js}^{i}_0, b_{js}^{i}_0, t_{s}^{ir}_0, (i,j=1,2,...,n), (r,s=1,2,...,m), and the a_{js}^{i}, b_{js}^{i}, t_{s}^{ir} are constant over time (any technological progress in the economy is excluded and spatial trading relationships are stable);
- all the productive processes exhibit constant returns to scale so that the a_{js}^{i} and the b_{js}^{i} do not change with respect to variations of output level $x_{js}(k)$, for all i, j and s;
- there is a one period lag between the capital goods acquisition and their utilisation, for all the acquiring sectors and for every capital good.

Restricting the analysis to an economy without international exchanges, the equilibrium relation between demand and supply for commodity i produced in region r can be written in matrix form as follows

$$x^r(k)=\sum_{s=1}^{m} T_s{}^r\{A_s\, x_s(k) + B_s[x_s(k+1)-x_s(k)]\} \qquad (5)$$

where $x^r(k)$, $x_s(k)$ and $x_s(k+1)$ are, respectively, the vectors representing the outputs produced by the n sectors in region r during period k and in region s during periods k and k+1; A_s is the nxn matrix of current input coefficients $a_{js}{}^i$ related to region s; B_s is the nxn matrix of capital input coefficients $b_{js}{}^i$ related to region s; $T_s{}^r$ is the nxn diagonal matrix of interregional trade coefficients from region r to s for all the n commodities:

$$T_s{}^r = \left|\begin{matrix} t_s{}^{1r} & 0 & \cdots & \cdots & 0 \\ 0 & t_s{}^{2r} & 0 & & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & t_s{}^{nr} \end{matrix}\right|$$

Expression (5) may be interpreted as an equilibrium relation between production of commodity i in region r and (demand as a current input) + (investment demand) for the commodity i in region r. From (1), (2), (3) and (4) equation (5) can be expressed in a more compact form as follows:

$$x(k) = TAx(k) + TB[x(k+1)-x(k)] \qquad (6)$$

where x(k) and x(k+1) are the vectors representing the outputs produced by all the sectors in all the regions during the periods k and k+1, whereas T, A and B are nmxnm matrices defined by the following relations:

$$T = \left|\begin{matrix} T_1{}^1 & T_2{}^1 & \cdots & \cdots & T_m{}^1 \\ T_1{}^2 & T_2{}^2 & \cdots & \cdots & T_m{}^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{matrix}\right|$$

$$\begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ T_1^m & T_2^m & \cdots & 0 & T_m^m \end{vmatrix}$$

$$A = \begin{vmatrix} A_1 & 0 & \cdots & \cdots & 0 \\ 0 & A_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & A_m \end{vmatrix}$$

$$B = \begin{vmatrix} B_1 & 0 & \cdots & \cdots & 0 \\ 0 & B_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & B_m \end{vmatrix}$$

By setting $U=(I-TA)^{-1}TB$, where I is the nmxnm identity matrix, system (6) can be transformed into:

$$x(k) = U[x(k+1)-x(k)]$$

Each element u_{ij}^{rs} of U represents the total amount of commodity i produced in region r required (directly and indirectly), as current and capital input, in order to increase by one unit the production of commodity j in region s.

3. EXISTENCE OF BALANCED GROWTH SOLUTIONS

Before discussing the existence of a balanced growth path for the dynamic multiregional input-output model (6), we need to recall some basic issues related to non negative systems. If a square matrix is non negative i.e., all of whose elements are non negative, then (Berman and Plemmons, 1979) its spectral radius is an eigenvalue, and in correspondence of this eigenvalue there exists a non negative eigenvector. Moreover, an irreducible matrix is characterised by exactly one (up to scalar multiplication) non negative eigenvector and this eigenvector is positive. In combinatorial terms a non negative matrix U irreducible if and only if for every pair (i, j) there exists a natural number q such that $u_{ij}^{(q)} > 0$. The characterisation of irreducibility has an interesting graph theoretic interpretation. To this aim we define the associated directed graph, G(U), which consists of mn vertices g_1, g_2,....g_{mn} where an edge leads from g_i to g_j if and only if u_{ij}_0. A directed graph G is strongly connected if for any ordered pair (g_i,g_j) of vertices of G there exists a sequence of edges (a path) which leads from g_i to g_j. Therefore, since $u_{ij}^{(q)}> 0$ if and only if there exists a sequence of q edges from g_i to g_j, then a matrix U is irreducible if and only if G(U) is strongly connected. The forward in time solution of system (6), then, is related to the solution of the eigenproblem of U (Campisi et al., 1991). Since both $(I\text{-}TA)^{-1}$ and TB are non-negative, so is their product U. Moreover if U is irreducible, it satisfies the hypothesis of the Perron-Frobenius theorem (Campisi and La Bella, 1988): there exists a positive simple eigenvalue λ^* of U with an associated positive eigenvector v^*. Hence, if the Hawkins-Simon condition holds, the irreducibility of matrix U is a sufficient condition for the existence of a balanced growth solution. In order to explore the relations between the properties of U and the balanced growth solution of system (6), the equivalence between the following two eigenproblems can be observed (Szyld, 1985):

$$U\, v = \lambda v \tag{7}$$

$$Hu = \mu TBu \tag{8}$$

where H = I-TA+TB and the eigenvectors of (8) are solutions at (6). Then, problems (7) and (8) share their eigenvectors and the following eigenvalue condition $\mu_i = (\lambda_i + 1) / \lambda_i, \forall i$ holds. It follows that in the long run all sectors in all regions grow up at the same rate $\mu^*=(\lambda^*+1)/\lambda^*$ depending from dominant eigenvalue λ^* and proportions among them are established according to the components of eigenvector v^*. Campisi and La Bella (1988) proved that U is irreducible if and only if the union graph G(T)"G(A)"G(B) is strongly connected. It follows that in a multiregional system where the Hawkins-Simon condition is satisfied and each sector in the economy utilises at least one capital good in its productive process, the strongly connection of the graph G(T)"G(A)"G(B) is a sufficient condition for the existence of a balanced growth solution. Observe that if the graph G(T)"G(A)"G(B) is strongly connected, each sector of the economy requires, directly or indirectly, either some current flow or some capital input from all the other sectors operating in the same and in the other regions. This condition is less restrictive than the assumption of irreducibility on matrix TA or TB. The irreducibility of U is, however, only a sufficient condition for the existence of a balanced growth path. In fact, it is possible to show (Campisi and La Bella, 1988) that a balanced growth solution can exist also in the case of a reducible U, i.e., when the system is divided into two or more groups of sectors in such a way that the sectors of some group do not need, neither directly nor indirectly, any current or capital inputs from some other group of sectors.

4. THE COMPONENTS OF MULTIREGIONAL GROWTH

The computation of the balanced growth path, however, often is an hard task: since U is an nmxnm matrix, it may happen that the matrices dimension is too large to be comfortably handled by standard methods. Thus, it may be useful to specify a technique for disaggregating the eigenproblem into smaller ones. Meyer (1989) has recently shown that it is possible to uncouple a nonnegative irreducible matrix by using the concept of Perron complementation. In the sequel, we will follow the above framework, which can be summarised as follows. Firstly we compute the dominant eigenvalue of matrix U by means of standard techniques, than we decompose the original problem in smaller ones. Of each of these smaller problems, all related to the above unique dominant eigenvalue, we compute the dominant eigenvectors. Finally, we reaggregate the single components into the original one. The advantage of the proposed technique can be condensed in two arguments. The first is related to the complexity of the problem: large-scale multiregional I-O systems require enormous computing time and large memory requirements related to the computation of the eigenvector components. The above procedure avoids this inconvenient and allows the use of conventional Personal Computers. The second advantage, depicted in a following section by means of a reduced scale exercise, allows to identify the sectoral and regional components of growth. In addition, it allows to work at a predetermined and/or wished regional or sectoral level of detail avoiding to maintain a large amount of data but preserving all the necessary linkages. Matrices and casual relations are therefore condensed into limited information. In spite of this advantages we need some technical jargon whose details are given in the sequel. In details, with reference to the multiregional dynamic I-O scheme, the Perron complementation allows to derive a sequence of smaller matrices having the following properties:

- each regional matrix is non negative and irreducible, so that it has a unique Perron-Frobenius eigenvector;
- each regional matrix has the same dominant eigenvalue of U (so preserving at regional level the overall growth properties);
- it is possible to determine the Perron-Frobenius eigenvectors of the regional matrices completely independently of each other;
- each regional matrix resumes the long run information related to the region under consideration;
- it is possible to couple these dominant eigenvectors back together in order to produce the long-run components associated to the multiregional operator U.

Therefore, it is possible to uncouple the computation of this trajectory into smaller problems. In this way the long run sector components for each regional production can be separated from the multiregional model. Thus, let D_{rr} (r = 1,...,m) be m regional matrices of order n associated to the multiregional growth matrix U. Each matrix will be non negative and irreducible with the same dominant eigenvalue of matrix U and a unique associated eigenvector (Meyer, 1989). Therefore, it is possible to couple the D_{rr} eigenvectors back together to produce the eigenvector for the original matrix U so allowing a structural analysis of long run growth rate and production mix. With reference to model (6), the multiregional growth operator U is assumed irreducible and its right eigenvector $v^{*}> 0$ appropriately normalised (the sum of its nm components equal to 1). Moreover, let U(nm,nm) be partitioned as a matrix with nxn square diagonal blocks:

$U_{1,1}$......$U_{1,n}$	$U_{1,n+1}$......$U_{1,2n}$	......	$U_{1,(m-1)*(n+1)}$......$U_{1,mn}$
....	....		....
$U_{n,1}$......$U_{n,n}$	$U_{n,n+1}$......$U_{n,2n}$	......	$U_{n,(m-1)*(n+1)}$......$U_{n,mn}$

U =

$U_{n+1,1}$......$U_{n+1,n}$ .... $U_{2n,1}$......$U_{2n,n}$	$U_{n+1,n+1}$.....$U_{n+1,2n}$ .... $U_{2n,n+1}$.....$U_{2n,2n}$		$U_{n+1,(m-1)*(n+1)}$......$U_{n+1,mn}$ .... $U_{2n,(m-1)*(n+1)}$......$U_{2n,mn}$
....	....		....
$U_{(m-1)*(n+1),1}$...... $U_{(m-1)*(n-1),n}$ .... $U_{mn,1}$......$U_{mn,n}$	$U_{(m-1)*(n+1),n+1}$...... $U_{(m-1)*(n-1),2n}$ .... $U_{mn,n+1}$.......$U_{mn,2n}$	.... 	$U_{(m-1)*(n+1),(m-1)*(n+1)}$...... $U_{(m-1)*(n-1),mn}$ .... $U_{mn,(m-1)*(n+1)}$......$U_{mn,mn}$

=

U_{11}	U_{12}	......	U_{1m}
U_{21}	U_{21}		U_{2m}
....	....		....
U_{m1}	U_{m2}	......	U_{mm}

(9)

For each region r, let U_r denote the principal block sub-matrix of U obtained by deleting its r-th row and its r-th column from U and U_{r*} and U_{*r} respectively given by:

$$U_{r*}=(U_{r,1}\ U_{r,2}\ \ldots\ldots\ U_{r,r-1}\ U_{r,r+1}\ \ldots\ldots\ U_{r,m}) \quad (10)$$

and

$$\begin{vmatrix} U_{1,r} \\ \ldots\ldots \end{vmatrix}$$

$$U_{*r} = \begin{vmatrix} U_{r-1,r} \\ \\ U_{r+1,r} \\ \ldots\ldots\ldots \\ U_{m,r} \end{vmatrix} \qquad (11)$$

In (10) U_{r*} represents the r-th row of blocks in (9) with $U_{r,r}$ removed and in (11) U_{*r} the r-th column of blocks with $U_{r,r}$ removed.

By means of (10) and (11) we can therefore provide the following :

DEFINITION 1. For a given region r, it is possible to resume all the long run information related to region r itself by means of the *regional complement matrix* of the block matrix U_{rr} in U defined as:

$$D_{rr}= U_{rr}+U_{r*}(\lambda^{*}I-U_{r})^{-1}U_{*r}$$

D_{rr} is a square non negative irreducible matrix of order n with dominant eigenvalue λ^* . Moreover if v^* is partitioned into its regional components:

$$v^{*} = \begin{vmatrix} v_1^{*} \\ v_2^{*} \\ \ldots\ldots\ldots \\ v_m^{*} \end{vmatrix}$$

then

$$D_{rr}\, v_r^{*}= \lambda^{*}\, v_r^{*}$$

so that each D_{rr} with r=1,2,.......,m, shares the dominant eigenvalue λ^* with an associated positive eigenvector $v_r^*>0$ (Meyer, 1989). This allows to state the following

DEFINITION 2. For a given region r, the *regional eigenvector* of $D_{r,r}$ is defined as:

$$p_r = \frac{v_r^*}{e^T v_r^*} = \frac{v_r^*}{\xi_r}$$

where $e^T = (1,1,....1)$. p_r represents the r-th segment of v^* normalised through the scalar $\xi_r = e^T v_r^*$ called the *regional coupling factor* . The significance of this factor is straightforward: it allows to measure and scale the growth of region r in comparison to the national one.

Then the sectoral components of the r-th region are given by the elements of eigenvector p_r and, since v^* can be written as a linear combination of the $D_{r,r}$ dominant eigenvectors with weights $\xi_{r,}$ grow up at the same rate $\mu^* = (\lambda^*+1)/\lambda^*$:

$$v^* = \begin{vmatrix} \xi_1 p_1 \\ \xi_2 p_2 \\ .... \\ .... \\ \xi_m p_m \end{vmatrix} \qquad (12)$$

Relation (12) decomposes the elementary factors of the growth process. In the long run, the sector components of each region r are given by the components of the eigenvector p_r with dimension n, whereas proportions among them are given according to the scalar weights ξ_r. Then (12) uncouples the multiregional eigenvector problem through its regional complements. Notice that on one hand the above measures cannot easily derived by means of the conventional approach and on the other hand they can be used in connection with sensitivity measures of multiregional Input-Output models (see for instance, Campisi and La Bella, 1990 and Campisi et al., 1990). In addition, if U is partitioned into m levels (so yielding m regional complements matrices), then all regional eigenvectors p_r can be determined independently and then combined in order to construct back the multiregional eigenvector v^*. This allows the computation of the multiregional stable production mix since the set of scalar weights

$$\xi^* = \begin{vmatrix} \xi_1 \\ \xi_2 \\ \cdots\cdots \\ \cdots\cdots \\ \xi_m \end{vmatrix}$$

is the eigenvector of the non negative and irreducible coupling matrix C(m,m) with dominant eigenvalue λ^*, whose entries are given by $C_{rs} = e^T U_{r,s} p_s$. Thus, to produce the normalised Perron vector v^* for the original nonnegative matrix U it is necessary to calculate firstly the dominant eigenvalue λ^* followed by the regional eigenvectors p_r and scalar weights ξ_r.

Notice that, to optimise the computational requirements, a balancing act may be performed when uncoupling the multiregional eigenvector problem through its regional complements. As the number of regions m largely increases, the partition of U becomes finer and the sizes of regional complements matrices become smaller, thus making easier to determine each regional eigenvector $p_{r,}$ even if the order of the matrix inversion becomes larger. At the same time, however, the size of the coupling matrix C becomes larger, thus making it more difficult to determine each regional coupling factor p_r. For these reasons, for very large problems it might be necessary to choose the partition which best suits the needs of the desired application. Furthermore the same procedure can be used to find the long-run components of growth of group of regions or to focus the direct and indirect effects of changes in selected sectors.

5. THE COMPUTER PROGRAM STRUCTURE

In this section a brief description of the related computer program using MATLAB is given. The program is divided in the following logic blocks as shown in Figure 1:

a) Input files and preliminary data

A directory contains a total of 3 files representing the necessary data base. The data base:

- Matrix T, Interregional commodity flows coefficients
- Matrix B, Output / Marginal capital ratios
- Matrix A, Technological coefficients

Given these matrices, the computer program calculates directly the matrix U and his dominant eigenvalue λ^*.

b) Selection of analysis

In this phase the program asks to choose between a regional analysis and a sectoral analysis. In this way it is possible both to select one or more regions and to select one or more sectors. The sectoral analysis can be performed both at national and regional level.

c) Matrix D definition and calculation of the related eigenvalue

The program allows to determine the growth matrix of the selected problem and calculate the related dominant eigenvector associated with the dominant eigenvalue previously calculated in accordance with the scalar normalisation factor.

d) Output files

The program provides a file that contains the results of the application.

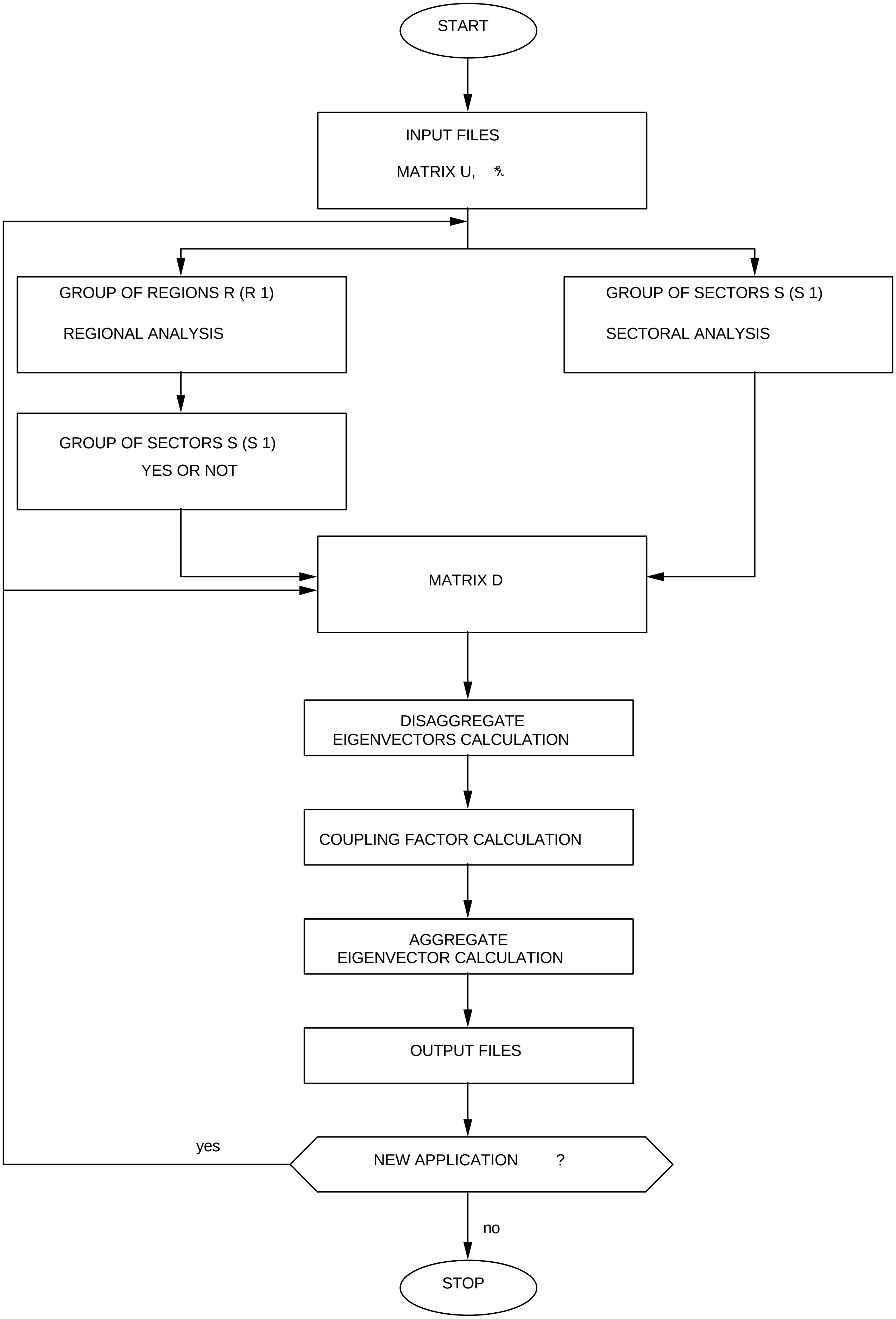

Fig. 1 - Overall computational process

6. A NUMERICAL APPLICATION TO THE DUALISTIC GROWTH OF ITALY

In this section the case of a two-level partition is presented. Let R be the set of the regions partitioned in two groups of regions G_1 and G_2 with G_1"G_2=R and G_1(G_2=_. Then, U can be partitioned as

$$U = \begin{vmatrix} U_{11} & U_{12} \\ \hline U_{21} & U_{22} \end{vmatrix} \qquad (13)$$

The multiregional eigenvector associated to the dominant eigenvalue λ^* is given by

$$v^* = \begin{vmatrix} \xi_1 \, p_1 \\ \xi_2 \, p_2 \end{vmatrix} \qquad (14)$$

where p_1 and p_2 are respectively the regional eigenvectors of

$$D_{11} = U_{11} + U_{12} \, (\lambda^* I - U_{22})^{-1} U_{21} \quad \text{and} \quad D_{22} = U_{22} + U_{21} \, (\lambda^* I - U_{11})^{-1} U_{12}$$

Moreover, since ξ_1 and ξ_2 are the two components of eigenvector of the coupling matrix

$$C = \begin{vmatrix} e^T U_{11} \, p_1 & e^T U_{12} \, p_2 \\ e^T U_{21} \, p_1 & e^T U_{22} \, p_2 \end{vmatrix}$$

the regional coupling factors ξ_1 and ξ_2 are given by (Meyer, 1989)

$$\xi_1 = \frac{e^T U_{12} \, p_2}{\lambda^* - e^T U_{11} \, p_1 + e^T U_{12} \, p_2} \quad \text{and} \quad \xi_2 = 1 - \xi_1$$

In order to implement the model proposed in this section we need to build up the biregional matrix U. In our numerical application the two groups of regions are the North of Italy (which comprises the following administrative regions: Piemonte, Val D'Aosta, Lombardia, Liguria, Trentino Alto Adige, Veneto, Friuli Venezia Giulia, Emilia Romagna, Toscana, Marche, Umbria, Lazio) and the South of Italy (which comprises the following administrative regions: Abruzzi, Molise, Campania, Puglia, Basilicata, Calabria, Sicilia, Sardegna). The regional economies are separated in the following 12 sectors (into brackets the numbers of corresponding branches of ISTAT classification at 1982 are reported):

1. Agriculture, forestry and fishing (1 - 5)
2. Energy (6 - 16)
3. Ferrous and non-ferrous metals (17 - 19)
4. Non-metal minerals (20 - 23)
5. Food, beverages and tobacco (40 - 54)
6. Chemical and pharmaceutical products (24 - 27)
7. Mechanical, vehicles, textiles and other manufacturing (28 - 39; 55 - 68)
8. Construction (69 - 70)
9. Trade, hotels, restaurants, scrap (71 - 74)
10. Transportation and communication (75 - 81)
11. Credit, finance and insurance (82 - 83)
12. For-sale and not-for-sale services (84 - 92)

As far as current input coefficients are concerned, a national 92-branch intersectoral flows table for Italy at 1985 and a 6-region, 12-sector intersectoral flows table for Italy at 1982 are available. Aggregating the former into a 12-sector table and the latter into a 2-regions, 12-sector one, and updating the 1982 biregional distribution of the intersectoral flows to 1985, it is possible to estimate the current input coefficients for the North and the South of Italy at 1985 as reported in Table 1 whereas the interregional trade coefficients (Campisi et al., 1991) aggregated at biregional level are reported in Table 2.

It is well known the difficulty in the computation of capital stocks. With reference to the Italian case, the results reported in Campisi and Nastasi (1989) are condensed in Table 3 where capital input coefficients for North and South are reported. Then, consider the Italian North-South economy at 1985 whose matrix U(nm,nm; n=12, m=2) with dominant eigenvalue $\lambda^*= 2.0196$ (the equilibrium growth factor μ^* is equal to 1.495), is partitioned in four sub-matrices U_{NN}, U_{NS}, U_{SN} and U_{SS} as shown in (13) where 1=N=North and 2=S=South. Detailed U tables are available from the authors. Then, from the above partition, the regional

Table 1 - Current input coefficients

North

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.1860	0.0000	0.0001	0.0019	0.4053	0.0044	0.0089	0.0005	0.0104	0.0005	0.0001	0.0014
2	0.0284	0.6430	0.1391	0.1324	0.0296	0.0967	0.0219	0.0140	0.0319	0.0940	0.0049	0.0260
3	0.0008	0.0008	0.3396	0.0198	0.0057	0.0051	0.0719	0.0474	0.0024	0.0005	0.0000	0.0008
4	0.0005	0.0005	0.0134	0.1577	0.0061	0.0270	0.0053	0.1424	0.0020	0.0003	0.0000	0.0009
5	0.0925	0.0000	0.0000	0.0000	0.1674	0.0161	0.0049	0.0000	0.0482	0.0016	0.0000	0.0052
6	0.0421	0.0055	0.0166	0.0355	0.0120	0.3767	0.0460	0.0185	0.0042	0.0019	0.0008	0.0144
7	0.0076	0.0171	0.0270	0.0533	0.0360	0.0507	0.3143	0.1499	0.0597	0.0569	0.0104	0.0434
8	0.0003	0.0086	0.0038	0.0068	0.0018	0.0019	0.0023	0.0373	0.0039	0.0190	0.0054	0.0323
9	0.0410	0.0042	0.1502	0.0732	0.0627	0.0646	0.0721	0.0364	0.0668	0.0491	0.0089	0.0210
10	0.0090	0.0109	0.0557	0.0568	0.0336	0.0398	0.0340	0.0362	0.0306	0.1100	0.0182	0.0172
11	0.0110	0.0023	0.0084	0.0058	0.0084	0.0060	0.0109	0.0125	0.0145	0.0178	0.6981	0.0101
12	0.0046	0.0059	0.0250	0.0388	0.0204	0.0515	0.0365	0.0452	0.0594	0.0509	0.2111	0.0756

South

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.1429	0.0000	0.0001	0.0016	0.3307	0.0029	0.0116	0.0004	0.0074	0.0004	0.0001	0.0012
2	0.0218	0.5800	0.1190	0.1068	0.0273	0.1084	0.0210	0.0121	0.0259	0.0628	0.0048	0.0213
3	0.0006	0.0007	0.3553	0.0159	0.0073	0.0049	0.0517	0.0410	0.0021	0.0004	0.0000	0.0007
4	0.0004	0.0005	0.0125	0.1271	0.0070	0.0322	0.0037	0.1242	0.0016	0.0002	0.0000	0.0008
5	0.0711	0.0000	0.0000	0.0000	0.1571	0.0093	0.0087	0.0000	0.0343	0.0010	0.0000	0.0048
6	0.0323	0.0050	0.0149	0.0286	0.0110	0.3893	0.0389	0.0162	0.0028	0.0013	0.0008	0.0118
7	0.0059	0.0147	0.0235	0.0429	0.0341	0.0418	0.3062	0.1308	0.0520	0.0412	0.0107	0.0371
8	0.0002	0.0077	0.0035	0.0055	0.0020	0.0014	0.0019	0.0327	0.0031	0.0198	0.0056	0.0255
9	0.0315	0.0037	0.1327	0.0590	0.0547	0.0586	0.0633	0.0318	0.0545	0.0347	0.0091	0.0166
10	0.0069	0.0098	0.0505	0.0458	0.0306	0.0358	0.0293	0.0314	0.0253	0.0750	0.0186	0.0141
11	0.0084	0.0020	0.0072	0.0047	0.0077	0.0060	0.0105	0.0109	0.0114	0.0120	0.7208	0.0078
12	0.0036	0.0051	0.0235	0.0313	0.0197	0.0406	0.0310	0.0394	0.0491	0.0361	0.1851	0.0605

Table 2 - Interregional trade coefficients* (diagonal element data)

North to North

Sector	
1	0.8054
2	0.7040
3	0.9042
4	0.9395
5	0.8658
6	0.8936
7	0.9098
8	1
9	1
10	1
11	1
12	1

South to South

Sector	
1	0.6817
2	0.7988
3	0.3521
4	0.7421
5	0.5365
6	0.5700
7	0.3630
8	1
9	1
10	1
11	1
12	1

*The interregional trade coefficients from South to North are obtained by subtracting the North to North matrix from the identity matrix; the interregional trade coefficients from North to South are obtained by subtracting the South to South matrix from the identity matrix.

Table 3 - Capital input coefficients**

North

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0020	0.0005	0.0013	0	0	0	0	0.0025	0	0	0	0
7	1.0304	0.5840	0.5772	0.6293	0.2502	0.4402	0.3046	0.1338	0.3072	0.9810	0.2622	0.2778
8	1.7629	1.8113	0.2591	0.2856	0.1485	0.1631	0.1404	0.0347	0.1768	0.6861	0.4070	3.3509

South

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0010	0.0004	0.0016	0	0	0	0	0.0016	0	0	0	0
7	0.5406	0.5920	0.7510	0.9180	0.3351	0.8380	0.7875	0.0900	0.3381	1.1555	0.5038	0.1958
8	0.9169	1.8201	0.3341	0.4130	0.1972	0.3078	0.3598	0.0231	0.1929	0.8012	0.7752	2.3420

**The capital coefficients for rows representing sector 2-6 and 9-12 are zeros.

Table 4 - Matrix $D_{NN}=D_{11}$

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0554	0.0409	0.0220	0.0226	0.0094	0.0154	0.0110	0.0070	0.0115	0.0383	0.0131	0.0506
2	0.4423	0.3717	0.1391	0.1519	0.0656	0.1008	0.0739	0.0288	0.0798	0.2728	0.1063	0.5482
3	0.4342	0.3524	0.1478	0.1614	0.0688	0.1081	0.0784	0.0311	0.0837	0.2832	0.1050	0.4937
4	0.3536	0.3477	0.0661	0.0726	0.0352	0.0441	0.0355	0.0108	0.0422	0.1569	0.0825	0.6167
5	0.0508	0.0393	0.0190	0.0205	0.0086	0.0139	0.0100	0.0043	0.0105	0.0352	0.0123	0.0511
6	0.2839	0.2262	0.1005	0.1096	0.0464	0.0737	0.0532	0.0215	0.0565	0.1903	0.0689	0.3078
7	2.4789	1.7531	1.0761	1.1739	0.4819	0.8055	0.5693	0.2395	0.5892	1.9356	0.6128	1.9019
8	1.8654	1.9099	0.2802	0.3087	0.1595	0.1775	0.1517	0.0383	0.1900	0.7342	0.4310	3.5220
9	0.4133	0.3361	0.1402	0.1530	0.0653	0.1025	0.0744	0.0296	0.0794	0.2688	0.0999	0.4721
10	0.2665	0.2267	0.0814	0.0890	0.0386	0.0588	0.0433	0.0166	0.0469	0.1612	0.0639	0.3398
11	0.2443	0.2075	0.0749	0.0819	0.0355	0.0541	0.0398	0.0154	0.0431	0.1481	0.0586	0.3103
12	0.3329	0.2843	0.1006	0.1100	0.0479	0.0726	0.0535	0.0205	0.0581	0.1999	0.0798	0.4286

Table 5 - Matrix $D_{SS}=D_{22}$

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0281	0.0389	0.0291	0.0340	0.0129	0.0303	0.0292	0.0045	0.0130	0.0458	0.0245	0.0314
2	0.4572	0.6807	0.4275	0.5226	0.2010	0.4618	0.4495	0.0482	0.2015	0.7200	0.4081	0.6097
3	0.0844	0.1282	0.0761	0.0931	0.0360	0.0819	0.0801	0.0085	0.0361	0.1295	0.0751	0.1186
4	0.1271	0.2356	0.0654	0.0804	0.0348	0.0652	0.0696	0.0059	0.0344	0.1344	0.1090	0.2837
5	0.0163	0.0234	0.0163	0.0199	0.0076	0.0177	0.0171	0.0019	0.0076	0.0269	0.0146	0.0196
6	0.0823	0.1235	0.0759	0.0928	0.0358	0.0819	0.0798	0.0085	0.0358	0.1283	0.0734	0.1121
7	0.5146	0.7732	0.6323	0.7731	0.2903	0.6934	0.6641	0.0733	0.2919	1.0227	0.5211	0.5631
8	0.9587	1.899	0.3542	0.4377	0.2081	0.3276	0.3812	0.0248	0.2037	0.8436	0.8110	2.4387
9	0.1045	0.1703	0.0808	0.0989	0.0393	0.0856	0.0852	0.0086	0.0392	0.1438	0.0919	0.1755
10	0.0763	0.1136	0.0519	0.0636	0.0259	0.0541	0.0549	0.0053	0.0258	0.0963	0.0665	0.1435
11	0.0800	0.1372	0.0540	0.0662	0.0270	0.0562	0.0571	0.0055	0.0268	0.1004	0.0696	0.1512
12	0.0961	0.1668	0.0627	0.0769	0.0316	0.0650	0.0664	0.0063	0.0314	0.1180	0.0836	0.1864

complement matrix $D_{NN}=D_{11}$ and $D_{SS}=D_{22}$ are presented in Tables 4 - 5. Each matrix above is a square, nonnegative irreducible matrix of order 12 with common dominant eigenvalue $\lambda^{*}= 2.0196$. Tables 4 - 5, therefore, give an interesting insight to the contribution of each productive sector in each region to the North and South growth. A more effective lecture can be obtained by means of p_1 and p_2 the dominant eigenvectors of D_{11} and D_{11} respectively. Each term gives a measure up to one of the single sectoral contributions at regional level. To obtain the national overall contribution, it is necessary to rescale these elements by means of the regional coupling factors ξ_1 and ξ_2. Therefore, the dominant eigenvector v^{*} for U can be obtained by (14):

$$v^{*} = \begin{vmatrix} 0.0064 \\ 0.0476 \\ 0.0477 \\ 0.0340 \\ 0.0057 \\ 0.0315 \\ 0.2931 \\ 0.1732 \\ 0.0454 \\ 0.0284 \\ 0.0261 \\ 0.0354 \\ \\ 0.0026 \\ 0.0411 \\ 0.0075 \\ 0.0100 \\ 0.0015 \\ 0.0074 \\ 0.0542 \\ 0.0715 \\ 0.0089 \\ 0.0063 \\ 0.0066 \\ 0.0079 \end{vmatrix} = \begin{matrix} [0.7744] \begin{vmatrix} 0.0082 \\ 0.0614 \\ 0.0616 \\ 0.0439 \\ 0.0074 \\ 0.0407 \\ 0.3785 \\ 0.2236 \\ 0.0586 \\ 0.0367 \\ 0.0337 \\ 0.0457 \end{vmatrix} \\ \\ [0.2256] \begin{vmatrix} 0.0114 \\ 0.1821 \\ 0.0333 \\ 0.0442 \\ 0.0066 \\ 0.0327 \\ 0.2403 \\ 0.3172 \\ 0.0396 \\ 0.0281 \\ 0.0294 \\ 0.0351 \end{vmatrix} \end{matrix} = \begin{vmatrix} \xi_1\, p_1 \\ \\ \xi_2\, p_2 \end{vmatrix}$$

Table 6 - Sectoral distribution of production

North

Sector	1985	Balanced Growth
1	0.0345	0.0082
2	0.0456	0.0614
3	0.0285	0.0616
4	0.0199	0.0439
5	0.0538	0.0074
6	0.0438	0.0407
7	0.2773	0.3785
8	0.0593	0.2236
9	0.1531	0.0586
10	0.0661	0.0367
11	0.0361	0.0337
12	0.1820	0.0457

South

Sector	1985	Balanced Growth
1	0.0749	0.0114
2	0.0513	0.1821
3	0.0250	0.0333
4	0.0156	0.0442
5	0.0458	0.0066
6	0.0262	0.0327
7	0.1223	0.2403
8	0.1006	0.3172
9	0.1586	0.0396
10	0.0640	0.0281
11	0.0214	0.0294
12	0.2943	0.0351

The regional coupling factors, given respectively by $\xi_1 = 0.7744$ and $\xi_2 = 0.2256$, confirm in the long run projection the historical dualism between the developed (North) and the underdeveloped (South) regions of Italy. At national level, each sector in North weights 77.44% whereas the corresponding in the South only 22.56%. Notice that sectoral distribution of production at 1985 presented in tables 6 is very different from the balanced growth one (Campisi *et al.*, 1991). In particular, the components associated to branch 12 (For-sale and not-for-sale services) of the North and especially of the South have a greater weight than the balanced ones. This is probably due to an oversized public services sector. On the contrary, the components associated to branch 2 (Energy) of the South and to branches 7 (Mechanical, autovehicles, textiles and other manufacturing) and particularly 8 (Construction) of both the regions are too "light" in comparison with the dynamic equilibrium ones. The other branches are less divergent from the balanced growth trajectories. So, with a modest computational effort it has been shown how to focus the long-run regional growth and appropriately manage the components affecting the multiregional growth. Notice that the potentialities of the approach have here only been sketched. When working at a more detailed level of analysis, it is possible to choose different levels of aggregation and focus the different components of growth. Furthermore, it can be effectively be used in the more traditional static I-O framework or to produce impact analyses.

7. CONCLUSIONS

The computation of the balanced growth of Input-output systems is an hard task. When dealing with multiregional schemes new theoretical and computational difficulties may be highlighted. Summing up traditional and new issues, in this paper we have proposed some key results to manage the difficulties deriving from the multiregional case. In particular, we have focused the problem of reducing the difficulties arising from the computation of the eigenvectors components associated to regional and sectoral growth. In addition, proportions among the growth components of each region are provided. The procedure permits the reduction of the computational complexity by allowing, at the meantime, an explanatory meaning of the structural elements. The procedure has been tested to analyse the biregional (North-South) growth of the Italian economy. The numerical exercise has been used to give light to the numerical approach and discuss some problems arising from the structure of the Italian economy.

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