Angular Measure for Matrix Similarity and Hadamard-multiplicative Generalization of RAS Method

Topic: Mathematical Treatments of Input-Output Relationships
Author: Vladimir MOTORIN

The aim of this study is to make more operational and to advance the notion of angular measure for structural similarity between target and initial matrices that has been introduced in the paper presented by the author at 22nd International Input-Output Conference in 2014. RAS multiplicative pattern of target matrix can be represented as Hadamardâ€™s product of the initial matrix and a factor matrix of the same dimension formed by matrix multiplication of column and row vectors of unknown parameters. Rather natural way to generalize RAS pattern is to abandon its biproportional framework in favor of multiparametrical approach with most common factor matrix. To satisfy row and column total constraints, it is proposed to disturb the elements of a factor matrix in some minimalistic manner. To this end, the notion of a homothetic ray for vectorized initial matrix in system of relative coordinates in multidimensional Euclidean space has been introduced. Homothetic measure for matrix similarity is then defined as a shortest path from unknown parametrical point (determined by vectorized factor matrix) to homothetic ray. Further, angular measure for matrix similarity represents an angle between vectorized factor matrix and homothetic ray. Note that both measures are invariant with respect to a way chosen for matrix vectorization. Since angular measure is to be minimized, a mathematical programming problem with the quadratic fractional objective function (so-called Rayleigh quotient) and two sets of mutually dependent linear constraints arises. This nonlinear optimization model is fully represented in matrix notation, and its solution is obtained in analytical form, which is suitable for sensitivity analysis implementation in the cases when the constraints are assumed non-binding (so the paper contains a variety of mathematical details). Main advantage of the proposed method is its immanent flexibility so in practice it allows to generate much more compact factor distributions in comparison with RAS method. Finally, the method is quite applicable for updating supply and use tables with some negative entries. A chain of illustrative numerical examples is given.