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Forecasting inflation using input-output model. Sources of errors.

Preliminary results

Introduction

The presented considerations are an empirical attempt to verify the possibility of using input-output tables for modeling and, as a consequence, forecasting inflation. The conclusions help to understand the price formation mechanisms. The study consisted in an attempt to reproduce the historical values of the commonly used CPI, basing on the input-output price model. The procedure involves ex post simulations of input-output price model for an open economy, and then applying appropriate weights to calculate a macroeconomic deflator of household consumption. A sequence of simulations under various assumptions was done to identify the sources of inflation.

The proposed procedure is significantly different from the most frequently used methods of forecasting inflation, which describe the macroeconomic price indicators (with higher than annual frequency) using stochastic models. A precise description of the price formation process is crucial for decisions made by central banks, which is why the most intense work is mainly focused on studying the pass-through mechanisms of exchange rate fluctuations on domestic prices. Input-output models create more opportunities to conduct analyzes taking into account price changes in individual markets. That’s why i-o price models also serve as tools for investigating price transmission mechanisms. The main caveat is the deterministic nature of the i-o price model.

Empirical research on inflation is based on three dominant theoretical positions, namely monetary, demand and cost theory. Research on price sensitivity to external impulses is of great importance for inflation forecasting. Such analyzes, often commissioned or supported by central banks, focus on the examination of pass-through mechanisms of exchange rate fluctuations on domestic prices (Auer and Schoenle 2016, Pennings 2017). The interest of economists is also aroused by the mechanisms of transmitting other price relations, especially the impulses coming from the raw materials market, mainly crude oil (Nazliglu 2011). Most of them use time series analysis tools. In general, these models are based on the cost formula in less (Bekkers et al 2017, Zhang, Qu 2015) or more (Pennings 2017) direct way.

Applications of the input-output price model are slightly less popular. It is a model based strictly on the cost formula, where the initial impulse is a change in import prices (see eg Wu and others 2013, Aydogus and others 2017) or unit value added (eg Lee et al. 2000, Sharify and Sancho 2011). More often, input-output models serve as parts of more extensive tools (like CGE or multisectoral econometric models).

General model

Due to the variety of reasons for the increase in production costs, different types of cost inflation have been distinguished:

- wage inflation, when the wage increase occurs independently of the demand for labor,
- inflation caused by profits, when companies use their monopolistic position to raise prices regardless of the demand for their products,
- imported inflation, caused by the increase in prices of imported goods,
- inflation caused by the increase in taxes and other non-wage labor costs.

All these mechanisms are reflected in the input-output price model.

The price in the input-output model is presented as the sum of all costs of production of a unit of a good. In the matrix notation, the price equation can be presented in the following way:

$$\boldsymbol{p}^D = \boldsymbol{A}^{D\prime}\boldsymbol{p}^D + \boldsymbol{A}^{M\prime}\boldsymbol{p}^M + \boldsymbol{v}, \qquad (1)$$

where

$\boldsymbol{p}^D$ – it is a vector of domestic prices with elements that are output deflators,

$\boldsymbol{A}^D$ – is a matrix of direct intermediate consumption of domestic products per unit of output,

$\boldsymbol{A}^M$ – is a matrix of direct intermediate consumption of imported products per unit of output,

$\boldsymbol{p}^M$ – vector of import prices,

$\boldsymbol{v}$ – vector of unit value added [1].

Important assumption posed on the above model is the homogeneity of prices. The prices of product group (p_i) are the same regardless of the buyer.

In the above equation both matrices represent quantitative flows of intermediates (matrix A^D is often called a matrix of technical coefficients), therefore the production costs (i.e. the value of inputs) are obtained after multiplying these matrices by appropriate prices. For practical reasons (lack of information in terms of quantity), contractual prices of products at level 1 are assumed, which allows to assume that the quantities of flows are equal to their values. Under such assumption the matrix of cost coefficients can be treated like the matrix of technical coefficient (similarly with import intensity).

For calculation purposes, it is convenient to reduce the above model to the form:

$$\boldsymbol{p}^D = (\boldsymbol{I} - \boldsymbol{A}^{D\prime})^{-1}\boldsymbol{A}^{M\prime}\boldsymbol{p}^M + (\boldsymbol{I} - \boldsymbol{A}^{D\prime})^{-1}\boldsymbol{v}. \qquad (2)$$

In the above equation, domestic prices change under the influence of two exogenous factors: changes in import prices and changes in unit value added, while matrices are treated as

[1] From practical reasons it includes also taxes posed on intermediate products.

parameters (price multipliers). The matrix $\boldsymbol{\Pi}' = (\boldsymbol{I} - \boldsymbol{A}^{D\prime})^{-1}\boldsymbol{A}^{M\prime}$ is decisive for the strength with which domestic prices will react on changes in import prices. Similarly, changes in unit added value are transmitted into prices according to transposed Leontief inverse matrix $\boldsymbol{L}' = (\boldsymbol{I} - \boldsymbol{A}^{D\prime})^{-1}$. Thus, the impact of import prices on the prices of domestic products depends on the import intensity of individual branches, but also on the structure of intermediate flows between them.

The $\boldsymbol{p}^D$ vector can be used to determine various macroeconomic deflators, using the appropriate set of weights. Here, the household consumption deflator (P^H) is considered to be the equivalent of CPI, based on the Paasche principle. This means that the weights are the quantities of household consumption in year (t +1) expressed in prices of the year t, denoted in the following formula as C. The weighting formula contains three components: prices of domestic products, prices of imported products (both categories expressed in basic prices) and change in taxes on final products (T):

$$P^H = \left(\sum_{i=1}^{n} p_i^D \cdot \frac{C_i^D}{C^D}\right) \cdot \frac{C^D}{C} + \left(\sum_{i=1}^{n} p_i^M \cdot \frac{C_i^M}{C^M}\right) \cdot \frac{C^M}{C} + \frac{TB}{T} \cdot \frac{T}{C}, \qquad (3)$$

where

$$C = C^D + C^M + T.$$

TB is the value of taxes calculated in current prices and T is the value of taxes in fixed prices from the base year). Subscript i means product group, no subscription - macroeconomic value, superscript D - domestic products, superscript M - imported products, lack of such superscript means total supply (i.e. D + M).

Although the i-o table and CPI are based on the same primary data and produced by the same statistical office, it is not expected, that the price index obtained on the basis of (3) should be equal to CPI.

The sequence of calculations

The experiment consisted of several ex post simulations, which were varied by components of formulas 1 and 2. They reflected a series of forecasts, where the future is gradually revealed. Results of simulations are numbered with the numbers given in the superscripts. The starting point is a "naive" forecast, where the prices were not expected to change:

$$\boldsymbol{p}_{t+1}^{D1} = \boldsymbol{p}_t^D = (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{A}_t^{M\prime}\boldsymbol{p}_t^M + (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{v}_t = \boldsymbol{i}, \qquad (4)$$

Simulations no. 2, 3 and 4 assume that parameters remain constant at the level of period t, but the exogenous variables are "revealed" and take the value of t+1.

$$\boldsymbol{p}_{t+1}^{D2} = (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{A}_t^{M\prime}\boldsymbol{p}_{t+1}^M + (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{v}_t, \qquad (5)$$

$$\boldsymbol{p}_{t+1}^{D3} = (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{A}_t^{M\prime}\boldsymbol{p}_{t+1}^{M} + (\boldsymbol{I} - \boldsymbol{A}_t^{D\prime})^{-1}\boldsymbol{v}_{t+1}. \quad (6)$$

Last simulation assumes that the parameters (A matrices) are also revealed, so all values are taken from t+1.

$$\boldsymbol{p}_{t+1}^{D4} = (\boldsymbol{I} - \boldsymbol{A}_{t+1}^{D}{}')^{-1}\boldsymbol{A}_{t+1}^{M}{}'\boldsymbol{p}_{t+1}^{M} + (\boldsymbol{I} - \boldsymbol{A}_{t+1}^{D}{}')^{-1}\boldsymbol{v}_{t+1}. \quad (7)$$

Ultimately, let's define $\boldsymbol{p}_{t+1}^{D5}$ with elements:

$$P_{i,t+1}^{K} = \frac{X_{i,\,t+1/t+1}}{X_{i,\,t+1/t}}$$

where $X_{i,\,t+1/t}$ is the output in the year expressed in prices of t, and $X_{i,\,t+1/t+1}$ is the same output in current prices. Assuming the homogeneity of prices, $\boldsymbol{p}_{t+1}^{D4}$ should be equal $\boldsymbol{p}_{t+1}^{D5}$. In the study presented below this assumption doesn't hold.

All above price vectors, numbered from 1 to 5, were multiplied by the same set of weights, to get the households consumption deflators:

$$P^{Hk} = \left(\sum_{i=1}^{n} P_i^{Dk} \cdot \frac{C_i^D}{C^D}\right) \cdot \frac{C^D}{C} + \left(\sum_{i=1}^{n} \mathrm{P}_i^{\mathrm{M}} \cdot \frac{C_i^M}{C^M}\right) \cdot \frac{C^M}{C} + \frac{TB}{T} \cdot \frac{T}{C}, \quad k = 1,\dots,5 \quad (8)$$

Please note, that P^{H1} is not equal to 1, because of change in taxes. For this reason let us define $P^{H0} = 1$. Let us also define

$$P^{H6} = \frac{CB}{C}, \quad (9)$$

Where CB and C are the values of household consumption expressed in current and previous year prices respectively.

In this sequence of deflators, the original value of CPI should be denoted as P^{H7}.

Differences between consecutive deflators were then calculated as:

$$E^k = P^{Hk} - P^{Hk-1}, \; k = 1,\dots,7, \quad (10)$$

thus:

$$P^{H7} - P^{H0} = \sum_{k=1}^{7} E^k. \quad (11)$$

The last difference (11) might be interpreted form two points of view.

1) It's the ex post error of naïve forecast.
2) As $P^{H0} = 1$, it's the actual inflation (CPI – 1).

Thus, the procedure described above decomposes inflation (or error) into seven factors (sources of errors), which are:

1) Changes in taxes posed on consumer goods
2) Changes in import prices (homogenous)
3) Changes in unit value added
4) Changes in parameters of the model (A matrices)
5) Assumption of homogeneity of domestic prices
6) Structure of household consumption[2]
7) Difference between CPI and household consumption deflator derived from the input-output table.

Data and the results

The simulations required the tables of flows of domestic products and (separate) tables of flows of imported products expressed in current and previous year prices. The widest available statistical material enabling such a study can be found at Statistics Denmark. The sets of i-o tables cover the period 1966 – 2017[3]. CPI however, is published starting form 1980, that's why our study is limited. The tables used in the study represent the aggregation at the level of 69 product groups.

The results are presented on graphs below. Additionally, linear correlation coefficients were shown in table 1.

Linear correlation between the components

	E1	E2	E3	E4	E5	E6	E7
CPI-1	0,76	0,70	0,88	-0,21	0,15	-0,08	-0,03
E1		0,54	0,54	0,05	-0,03	-0,11	-0,21
E2			0,71	-0,36	0,09	-0,60	-0,48
E3				-0,56	0,13	-0,18	-0,25
E4					-0,17	0,22	0,23
E5						-0,21	0,17
E6							0,33

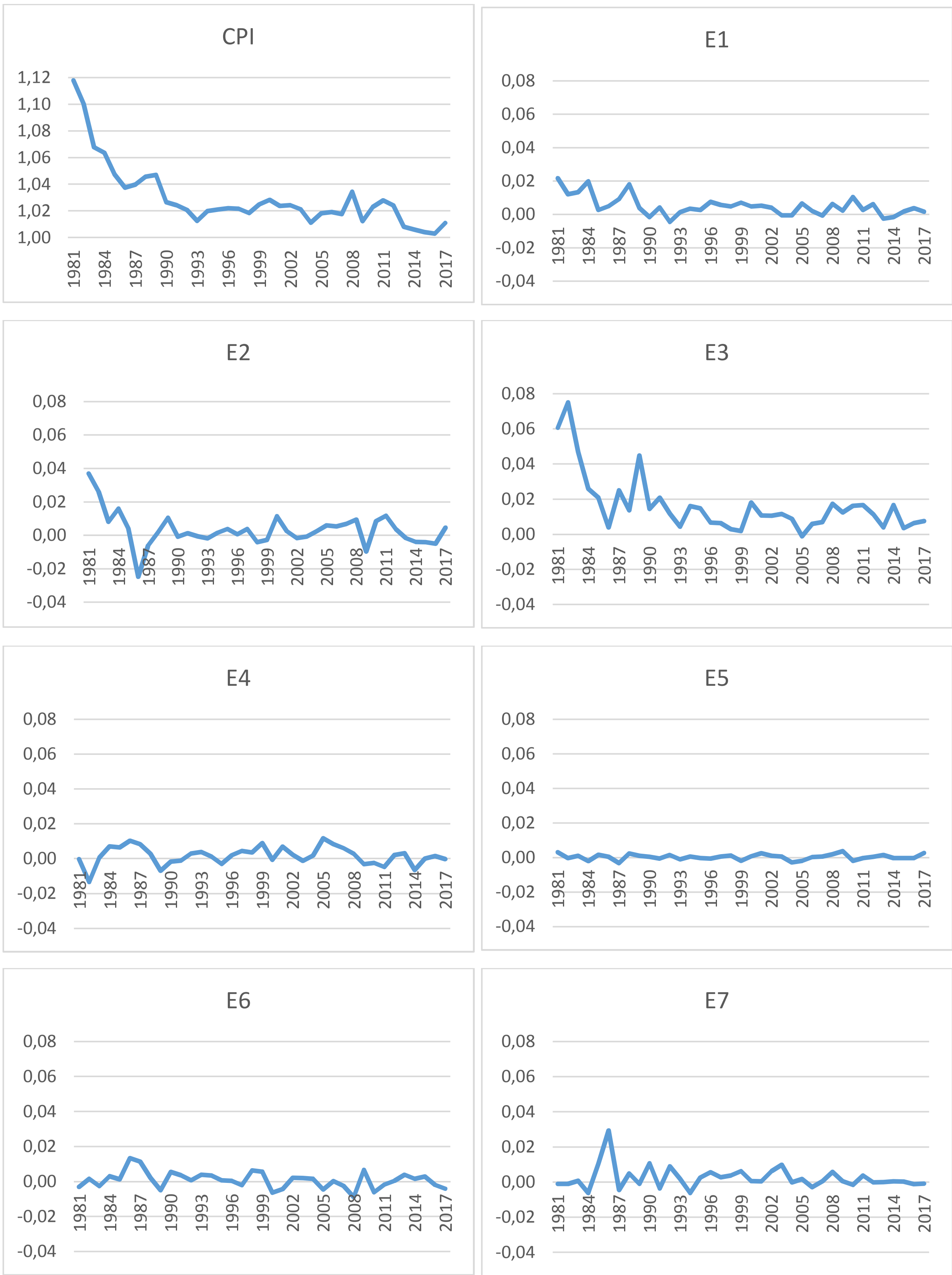
Source: own calculations based on Statistics Denmark data.

[2] the source of error is the intra-industry specificity of household spending. For example, in the case of agricultural products, household purchases include, for example, apples and bananas, but do not include rape or flax, which are mainly intermediate products. If the prices of apples and bananas go in different direction than the prices of rapeseed and flax, the consumers price index for agricultural products will be different than producers price index. The deflator of output will only show the average change.

[3] https://www.dst.dk/en/Statistik/emner/nationalregnskab-og-offentlige-finanser/produktivitet-og-input-output/input-output-tabeller

Dannish CPI and its decomposition.

The sum of E1..E7 equals to CPI-1



Source: own calculations based on Statistics Denmark data.

The first three components change in accordance with inflation, while other four seem to be rather stable over time, regardless of CPI. The main driving force for inflation is the unit value added (E3). The second important factor are the prices of imports (E2). These results confirm the price formation process reflected by the input-output model. Taxes (E1) are correlated with inflation and the role of this component is ambiguous; changes in tax rates are a cause of inflation, but inflation itself rises the tax revenues.

High correlation between E2 and E3 proves, that changes in prices of imports force the adjustment mechanisms influencing value added. One can suspect, that other adjustments relate to change in model parameters, i.e. A^D and A^M matrices (E4).

Small values of E5 mean that the assumption of homogeneity of domestic prices doesn't seem to be a significant source of errors, when projecting CPI. This is an argument encouraging the use of the model described in formula (1). The last two components (E6 and E7) have also (like E5) a "technical" character. Strong negative correlation between E2 and E6 may suggest the existence of a substitution effect, i.e. that structure of consumption depends on import prices.

Summary and conclusions

Although the presented study is generally empirical, practical application of the input-output model for forecasting inflation is not the aim of it. The authors consider it rather as a contribution to discussion on price formation and transmission mechanisms.

The study was limited to the example of Denmark, which resulted from the availability of statistical material. Thanks to the existence of a unified methodology for creating input-output tables used by Eurostat, the described procedure can be applied universally. However, the existing Eurostat data set limits the possibility of conducting such an experiments. In perspective, necessary i-o tables will certainly appear, and it can be expected that their detail will increase.

It seems, therefore, that the presented analysis is a good starting point for undertaking further activities in this area. In the light of the obtained results, the most interesting directions seem to be considering more components, e.g. elements of value added, separating matrices of parameters, exchange rate, and investigating interdependencies between them. Obtaining more empirical material in the form of time series should allow the use of more sophisticated statistical and econometric methods.

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